CORRECTION FOR PID CONTROLLER VALUE FOR BUS SUSPENSION MODEL WITH MATLAB

A. M. Shiddiq Yunus[1)]

Abstract: One important thing of design the real system is to make a simulation to find out the optimal and the best performance of the system. Bus suspension has been modeling for some control and simulation systems and this paper is aimed to find the better performance of bus suspension system with PID Controller. The main idea of this paper is to discuss about the correction for better result in controlling the bus suspension system using PID Controller through Simulink in Matlab program. The result shows that in this design, the PID controller zeroes are located at -4 and -27 on the real axis. The desired pole locations have been determined and are located at -362, -31, -1 -2-j12.3 and -2+j12.3. For the value of PID, it desired to put the gains for Ki, Kp, and Kd respectively: 11153000, 3065000, and 100261. From the plotted graphic, it can be seen that there is an approximate overshoot of 12% and a settling time of around 1 second.

Key word: PID controller, bus suspension, simulation and modelling.

I. INTRODUCTION

Designing an automatic suspension system for a bus turns out to be an interesting control problem. When the suspension system is designed, a 1/4 bus model (one of the four wheels) is used to simplify the problem to a one dimensional spring-damper system. This model meets the following requirements: the system is 4th-order, there are two pairs of complex conjugate poles, the system is linear and time-invariant, the system has a sub optimal response and system requires active-closed loop control to maintain stable and optimum control.

II. RESEARCH METHODOLOGY

Research methodology applies in this scientific paper is case study with real bus equation system and using approximation values for all used variables. Moreover, to assist the reliable result of this study, Matlab programming is applied, particularly for analyzing desired graphic of design system using Simulink in Matlab. The design system of bus suspension is seen such below:
A diagram of this system is shown below (figure 1).

1 Staf Pengajar Jurusan Teknik Mesin Politeknik Negeri Ujung Pandang

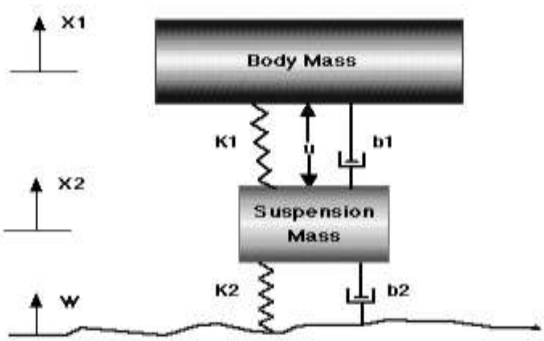

Figure 1: Model Of Bus Suspension System

We will define the equations, transfer function(s), transient Characteristics, etc, for the physical system of a Bus Suspension as the following:

Natural Behavior assumption of the System (system equations)
Where:

- Body mass (**M1**) = 2500 kg.
- Suspension mass (**M2**) = 320 kg.
- Spring constant of suspension system (**K1**) = 80,000 N/m.
- Spring constant of wheel and tire (**K2**) = 500,000 N/m.
- Damping constant of suspension system (**b1**) = 350 Ns/m.
- Damping constant of wheel and tire (**b2**) = 15,020 Ns/m.
- Control force (**U**) = force from the controller we are going to design.
- (**W**) The road disturbance in this problem will be simulated by a step input.
- (**X1-W**) the distance between suspension and the ground which is very difficult to measure because, the ground surface is always changed.
- (**X2-W**) the deformation of the tire which is negligible.
- (**X1-X2**) the distance between suspension and the body which we will use it as an output instead of (X1-W) in our problem.

We can write the dynamic equations from the Newton's law and picture above, as the following:

$$M_1\ddot{X}_1 = -b_1(\dot{X}_1 - \dot{X}_2) - K_1(X_1 - X_2) + U \quad \rightarrow (1)$$

$$M_2\ddot{X}_2 = b_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + b_2(W - \dot{X}_2) + K_2(W - X_2) - U \quad \rightarrow (2)$$

The transfer function (converting them into Laplace Model) of the formulations above is:

$$(M_1S^2 + b_1S + K_1)X_1(S) - (b_1S + K_1)X_2(S) = U(S) \quad \rightarrow (1)$$

$$M_2\ddot{X}_2 = b_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + b_2(W - \dot{X}_2) + K_2(W - X_2) - U \quad \rightarrow (2)$$

By compensating U(s) into equation (2), we obtaining:

$$(M_1S^2)X_1(S) + (M_2S^2 + b_2S + K_2)X_2(S) = (b_2S + K_2)W(S) \quad \rightarrow (2)$$

Now, we will use matrices to solve the two equations as below:

$$\begin{bmatrix} (M_1S^2 + b_1S + K_1) & -(b_1S + K_1) \\ M_1S^2 & M_2S^2 + b_2S + K_2 \end{bmatrix} \times \begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} \equiv \begin{bmatrix} U(S) \\ (b_2S + K_2)W(S) \end{bmatrix}$$

Obtaining determinant:

$$A = \begin{bmatrix} (M_1S^2 + b_1S + K_1) & -(b_1S + K_1) \\ M_1S^2 & M_2S^2 + b_2S + K_2 \end{bmatrix}$$

$$\Delta = \left((M_1S^2 + b_1S + K_1) \times (M_2S^2 + b_2S + K_2) - (M_1S^2) \times -(b_1S + K_1)\right)$$

$$\Delta = \begin{bmatrix} (M_1M_2S^4 + M_1b_2S^3 + M_1K_2S^2) + (M_2b_1S^3 + b_1b_2S^2 + b_1K_2S) \\ + (M_2K_1S^2 + b_2K_1S + K_1K_2) + (M_1b_1S^3 + M_1K_1S^2) \end{bmatrix}$$

Calculate the inverse of matrix A:

$$A^{-1} = \begin{bmatrix} M_2S^2 + b_2S + K_2 & (b_1S + K_1) \\ -(M_1S^2) & (M_1S^2 + b_1S + K_1) \end{bmatrix}$$

Simplifying it by multiplying with both side of equation:

$$\begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} \equiv \frac{1}{\Delta}\begin{bmatrix} (M_2S^2 + b_2S + K_2) \times U(S) + (b_1S + K_1) \times (b_2S + K_2)W(S) \\ -(M_1S^2) \times U(S) + (M_1S^2 + b_1S + K_1) \times (b_2S + K_2)W(S) \end{bmatrix}$$

In order to obtain the transfer function **G1(s)** we will put **W(s)** = 0 and we consider input **U(s)** only.

$$X_1(S) \equiv \frac{(M_2S^2 + b_2S + K_2) \times U(S) + (b_1S + K_1) \times (b_2S + K_2)W(S)}{\Delta}$$

$$X_2(S) \equiv \frac{-(M_1S^2) \times U(S) + (M_1S^2 + b_1S + K_1) \times (b_2S + K_2)W(S)}{\Delta}$$

$$X_1(S) \equiv \frac{(M_2S^2 + b_2S + K_2) \times U(S)}{\Delta} \quad \textit{and} \quad X_2(S) \equiv \frac{-(M_1S^2) \times U(S)}{\Delta}$$

$$G1(S) = \frac{X(S) - X2(S)}{U(S)} = \frac{(M_2S^2 + b_2S + K_2) - (-(M_1S^2))}{\Delta} = \frac{(M_1 + M_2)S^2 + b_2S + K_2}{\Delta}$$

The equation stated above is actual force equation only.

To obtain the transfer function **G2(s)** we will put **U(s)** = 0 and we consider input **W(s)** only.

$$X_1(S) \equiv \frac{(b_1 S + K_1) \times (b_2 S + K_2) W(S)}{\Delta} \equiv \frac{(b_1 b_2 S^2 + K_1 b_2 S + b_1 K_2 S + K_1 K_2) W(S)}{\Delta}$$

$$X_2(S) \equiv \frac{(M_1 b_2 S^3 + M_1 K_2 S^2 + b_1 b_2 S^2 + b_1 K_2 S + b_2 K_1 S + K_1 K_2) W(S)}{\Delta}$$

$$= \frac{(M_1 b_2 S^3 + (M_1 K_2 + b_1 b_2) S^2 + (b_1 K_2 + b_2 K_1) S + K_1 K_2) W(S)}{\Delta}$$

$$G2(S) = \frac{X_1(S) - X_2(S)}{W(S)}$$

$$= \frac{\left[b_1 b_2 S^2 + (K_1 b_2 + b_1 K_2) S + K_1 K_2\right] - \left[M_1 b_2 S^3 + (M_1 K_2 + b_1 b_2) S^2 + (b_1 K_2 + b_2 K_1) S + K_1 K_2\right]}{\Delta}$$

$$= \frac{-M_1 b_2 S^3 - M_1 K_2 S^2}{\Delta}$$

The equation above is disturbance force equation only.

Discussion

Characterize the transient and steady-state response of the system using Matlab

G1(s) = numU/denU (actuated force equation)

Matlab program:

```
m1=2500;
m2=320;
k1=80000;
k2=500000;
b1 = 350;
b2 = 15020;
det=[(m1*m2) (m1*(b1+b2))+(m2*b1) (m1*(k1+k2))+(m2*k1)+(b1*b2)
(b1*k2)+(b2*k1) k1*k2];
numU=[(m1+m2) b2 k2];
denU=det
Gp=TF(numU,denU)
```

The transfer function is imported to LTI view to examine the transient and steady state response of the system as can be seen below in figure 2.

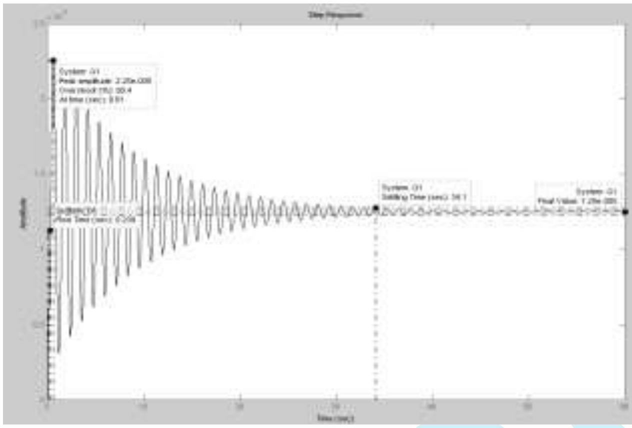

Figure 2. Transient And Steady State Response

Optimal Performance Parameters to be Achieved Using Closed-loop Control

In designing optimal parameters for our bus suspension model we have to take into consideration the passengers comfort. Optimum control can be achieved with an unlimited money source but this is unrealistic for a common bus. Therefore we determined the maximum overshoot should be less than 5% of the original step disturbance. The second important parameter is settling time, this should be less than 5 seconds. This would mean that as soon as the bus encounters a disturbance on the road the passengers would initially feel 5% of the total tire displacement from the ground and very small bus oscillations for a total of 5 seconds.

In figure 3 we simulated a closed loop system with our performance parameters entered into Matlab's Sisotool design tool box. The straight vertical line represents our settling time and 2 oblique lines is our maximum overshoot.

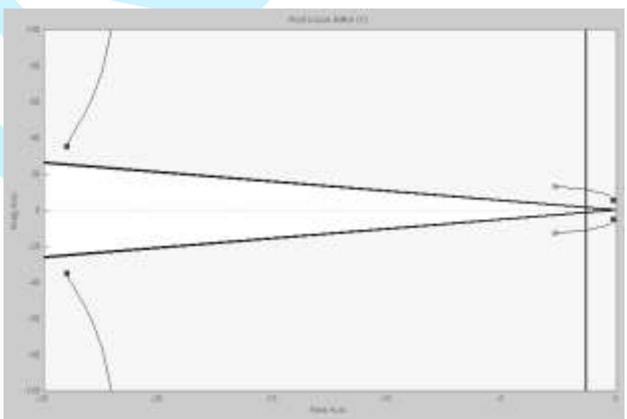

Figure 3. Closed Loop System With Design Constraints

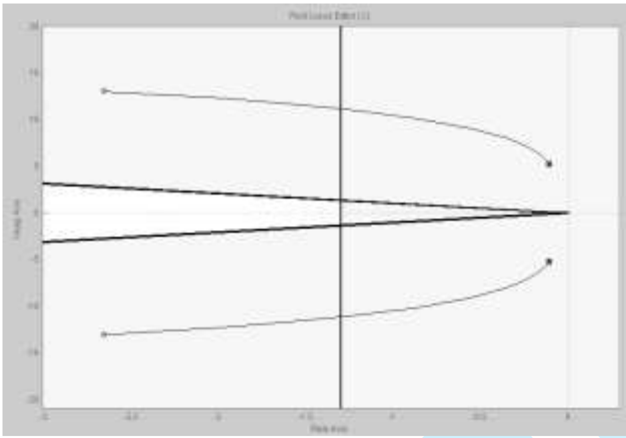

Figure 4. Zoom Up Of Dominate Poles In Closed Loop System With Design Constraints

In our closed loop system we have 2 poles close to the origin (the 2 poles are represented as red dots). It is the position of these 2 poles that are of the most concern, because they directly affect our system response times. Since they are so close to the origin they slow down the system a lot and cause the long settling time. The aim of our PID controller is to create a root locus where we can pull these poles as far left as possible. Pulling the poles to the left will also help satisfy our design parameters. The more our dominate poles are pulled towards the left of the design constraint lines the more they satisfy our parameters.

After examining the system and attempting to apply simple control methods we have decided to control our system with a PID controller. As we know that a PID controller is a type of compensator and we used it to reduce sensitivity to parameter variations and disturbances with improving steady-state error. Figure 8 shows the typical position of a PID controller placed within the control loop

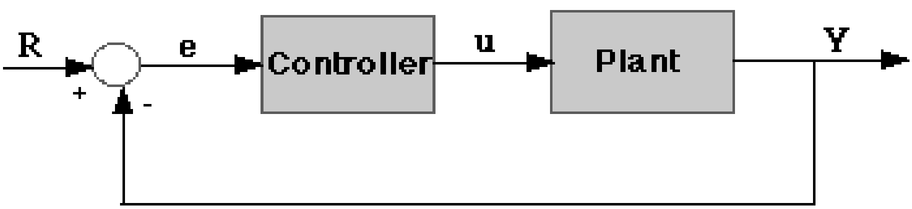

Figure 5. Typical Arrangement Of PID Control Loop

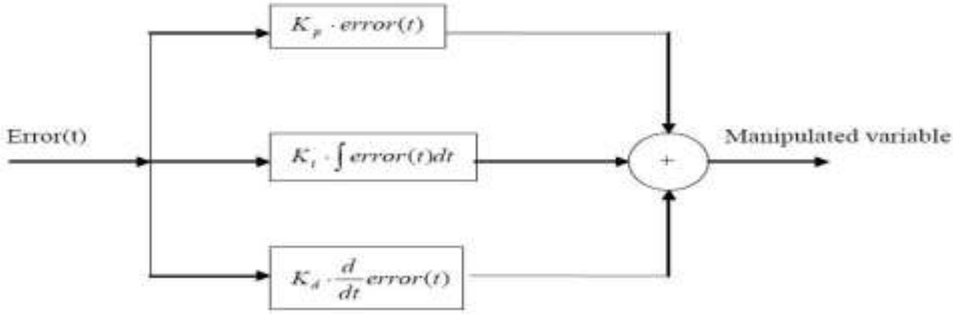

Figure 6. Elements Within A PID Controller

Where:

Kp = Proportional gain: increase the loop gain and reduce the sensitivity.

Ki = Integral gain: increase the order of the system and reduce steady-state error.

Kd = Derivative gain: stabilize the system.

The ideal transfer function of the PID controller looks like the following:

$$K_p+\frac{K_i}{s}+K_d s=\frac{K_d s^2+K_p s+K_i}{s}$$

From this equation we have two new zeros and one new pole at the origin added to our system. In choosing the position of the poles and zeros of the PID we chose to place zeros at -4 and -27 and a pole at zero. The position of these poles bends the root locus to a desirable position. Then we chose where we wanted our resultant characteristic poles of our compensated system on the branches of the root locus by altering the closed loop gain. We chose pole locations at -362, -31, -4, s+2-j12.3 and s+2+j12.3 which gave us a suitable response to a step input matching our design constraints

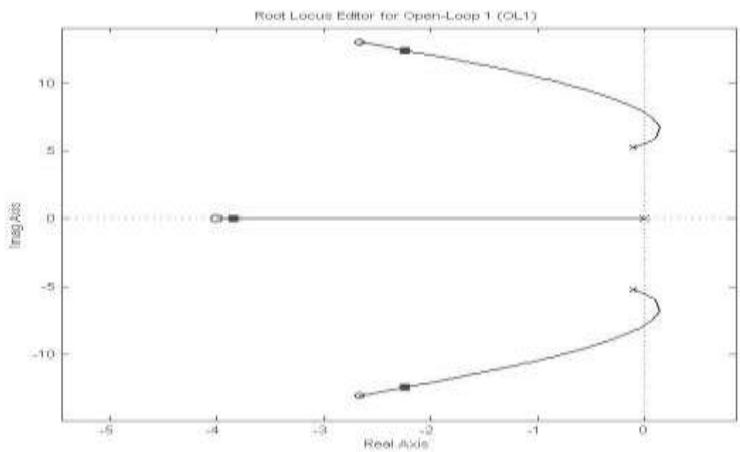

Figure 7. Pole Location Of Dominate Poles

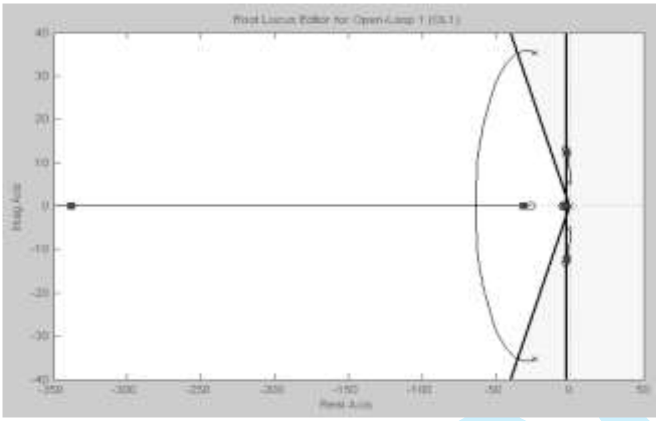

Figure 8. Pole Locations Of Desired System

From figure 7 and 8, the red dots represent the pole location of our required controlled system. Figure 9 displays the step input of our controlled system. The pole locations seem to satisfy our design criteria.

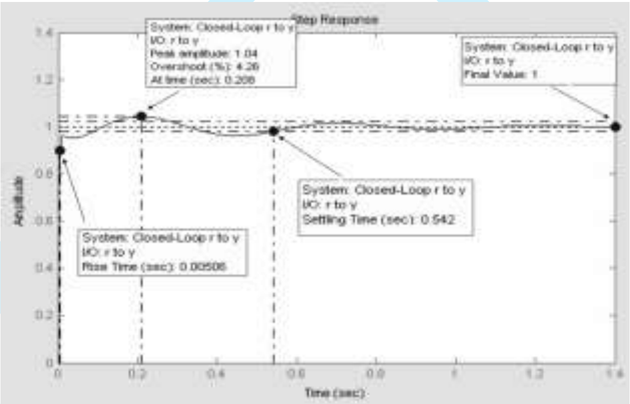

Figure 9. Step Response Of Desired Closed Loop Transfer Function

III. PID CALCULATIONS

There are many methods in tuning PID controller which include empirical and analytical methods. Empirical methodology involves manually manipulating PID gains to improve the response of the current system. Nichols Ziegler and Tyreus-Luyben are empirical methodologies which were designed to tune closed loop control system for the desired response. These methods are used to adjust parameters to correct steady-state errors and to obtain the desirable response.

The analytical method involves determining the values of the integral, derivative and proportional gain using rigorous calculations if the plant is known. This technique requires placing the closed loop poles at the desired locations to manipulate the root locus to improve the response of the current system design. The gains of the PID controller can therefore be calculated by comparing the characteristic equation of the current closed loop system to the desired characteristic equation.
The analytical method was used in this design since the plant has been defined and that the desired closed loop poles have been determined.

Analytical Calculations

The analytical method requires two constraints which include desired locations of the poles and the location of the zeroes and poles of the PID controller. In this design, the PID controller zeroes are located at -4 and -27 on the real axis. The desired pole locations have been determined and are located at -362, -31, -1 -2-j12.3 and -2+j12.3. Refer to figure 3 for pole placement diagram.
Closed loop gain with PID controller:

$$CLTF = \frac{C(s)}{R(s)} = \frac{\left(Kp + \frac{Ki}{s} + Kds\right)G_p(s)}{1 + \left(Kp + \frac{Ki}{s} + Kds\right)Gp(s)}$$

Where: Gp(s) is the transfer function of the plant, Kp is the proportional gain, Ki is the integration gain and Kd is the derivative gain.

Transfer function of the plant:

$$Gp(s) = \frac{2820\ s^2 + 15020\ s + 500000}{800000\ s^4 + 3.854 \times 10^7\ s^3 + 1.481 \times 10^9\ s^2 + 1.377 \times 10^9\ s + 4 \times 10^{10}}$$

Current closed loop system:

$$CLTF = \frac{\left(Kp + \frac{Ki}{s} + Kds\right) \times \frac{2820 s^2 + 15020 s + 500000}{800000 s^4 + 3.854 \times 10^7 s^3 + 1.481 \times 10^9 s^2 + 1.377 \times 10^9 s + 4 \times 10^{10}})}{1 + \left(Kp + \frac{Ki}{s} + Kds\right) \times \frac{2820 s^2 + 15020 s + 500000}{800000 s^4 + 3.854 \times 10^7 s^3 + 1.481 \times 10^9 s^2 + 1.377 \times 10^9 s + 4 \times 10^{10}}}$$

By simplifying the mathematical equation, the following characteristic equation was obtained.

$$s^5 + (3.525 \times 10^{-3} Kd + 48.175)s^4 + (3.525 \times 10^{-3} Kp + 18.78 \times 10^{-3} Kd + 1851.25)s^3$$

$$+ (18.78 \times 10^{-3} Kp + 0.625 Kd + 3.525 \times 10^{-3} Ki + 1721.25)s^2 +$$

$$(0.625 Kp + 18.78 \times 10^{-3} Ki + 50000)s + 0.625 Ki$$

Desirable characteristic equation calculation:

$$(s+362)\times(s+31)\times(s+4)\times(s+2-j12.3)\times(s+2+j12.3)$$

By expanding the equation, the following characteristic equation was obtained.

$$s^5 + 401s^4 + 1.454\times10^4 s^3 + 1.577\times10^5 s^2 + 2.166\times10^6 s + 6.97\times10^6$$

Determining the gains was done by comparing the current system characteristic equation and the desirable characteristic equation.

$$401 = 3.525\times10^{-3} Kd + 48.175$$

$$1.454\times10^4 = 3.525\times10^{-3} Kp + 18.78\times10^{-3} Kd + 1851.25$$

$$1.577\times10^5 = 18.78\times10^{-3} Kp + 0.625 Kd + 3.525\times10^{-3} Ki + 1721.25$$

$$2.166\times10^6 = 0.625 Kp + 18.78\times10^{-3} Ki + 50000$$

$$6.97\times10^6 = 0.625 Ki$$

The gains were calculated as the following:
Integration Gain (ki) = 11153000
Proportional Gain (kp) = 3065000
Derivative Gain (kd) = 100261

Analysis of Calculated Values

Transferring the calculated values into the simulink tool box we get the following step response. Seen in figure 10.

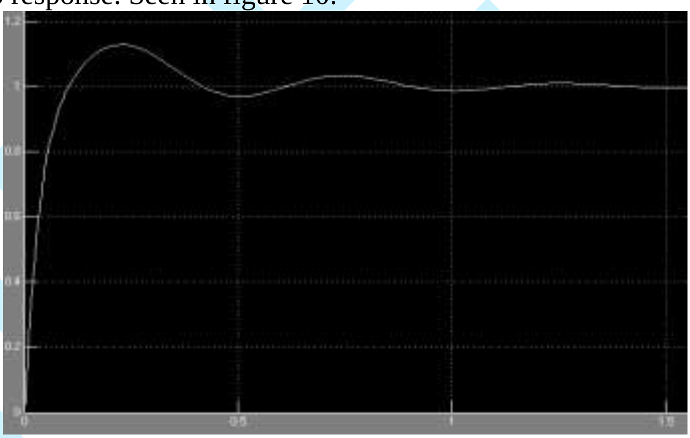

Figure 10. Simulink Response Of Compensated System

It can be seen there is an approximate overshoot of 12% and a settling time of around 1 second.

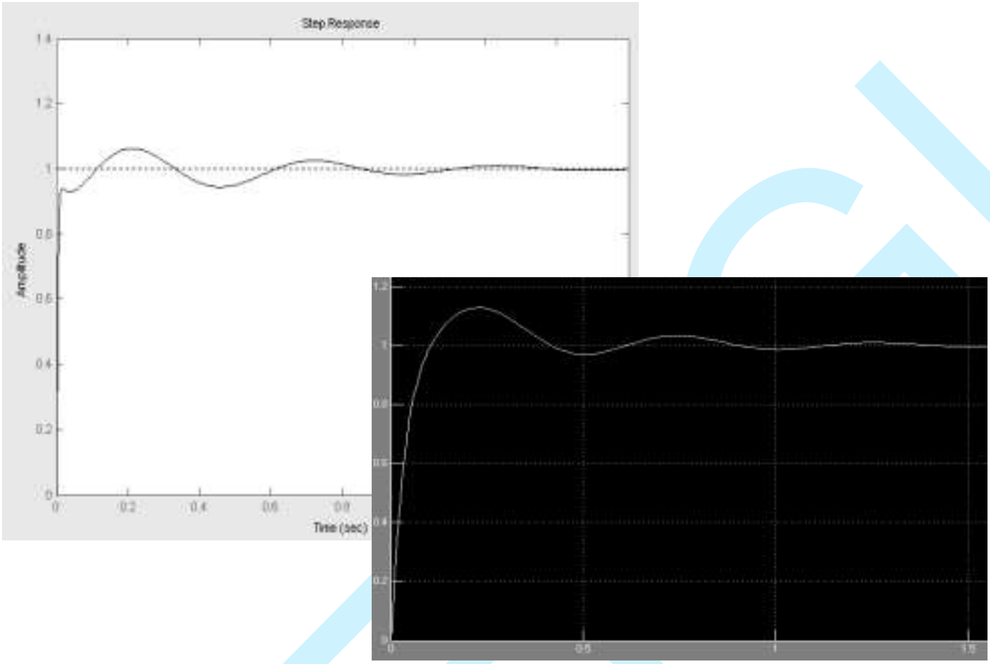

Figure 11. Side By Side Comparison. Left, Sisotool's Lti View. Right, Simulink Response

A side by side comparison of the sisotool model LTI response and our equivalent calculated simulink model reviles a slight difference in overshoot. The settling time and period is the same. They both peak at the same times but our overshoot percentage is slightly different. We wanted a 5% overshoot but our calculated values resulted in almost a 12 % overshoot.

We believe the cause of the overshoot difference is due to the transient spike in the first few seconds on the LTI view. We believe this is due to not sampling at a high enough rate, however we could weren't able to change the sampling time within sisotool. Even though our values didn't match our original design requirements we believe it is appropriate for a bus suspension system.

Analysing the disturbance

Another equation was acquired during the transfer function analysis. The referenced website refers to this as a disturbance equation so we decided to model it. After putting it through a step input we get the following waveform in figure 12.

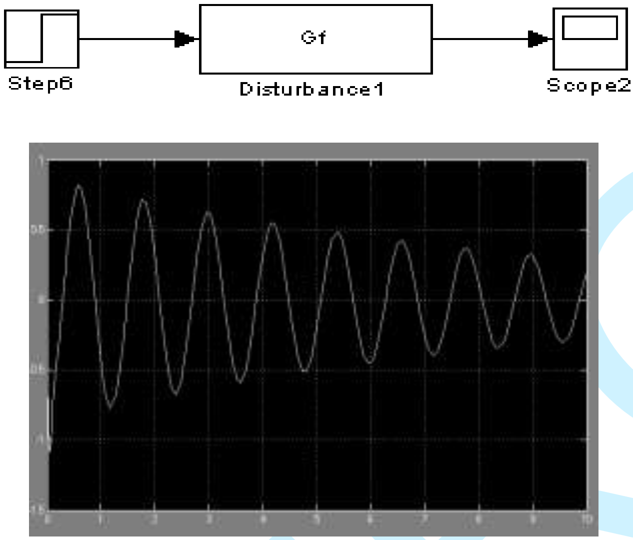

Figure 12. Characteristic Of Disturbance Equation

The disturbance equation seems to model what we believe to be a rocky road. We decided to see how our PID controlled system would handle the disturbance. We placed the disturbance into our control loop as shown in figure 13. To test the disturbance a step input is applied to the system, then after 2 seconds we applied a step input to the disturbance. The output is shown in figure 14.

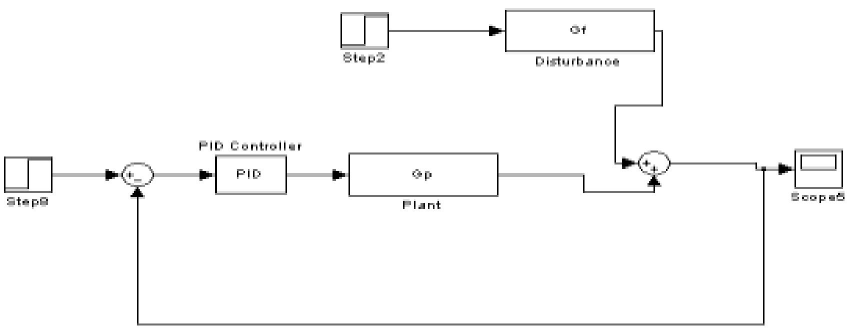

Figure 13. Simulink Diagram Of Out Disturbance Test

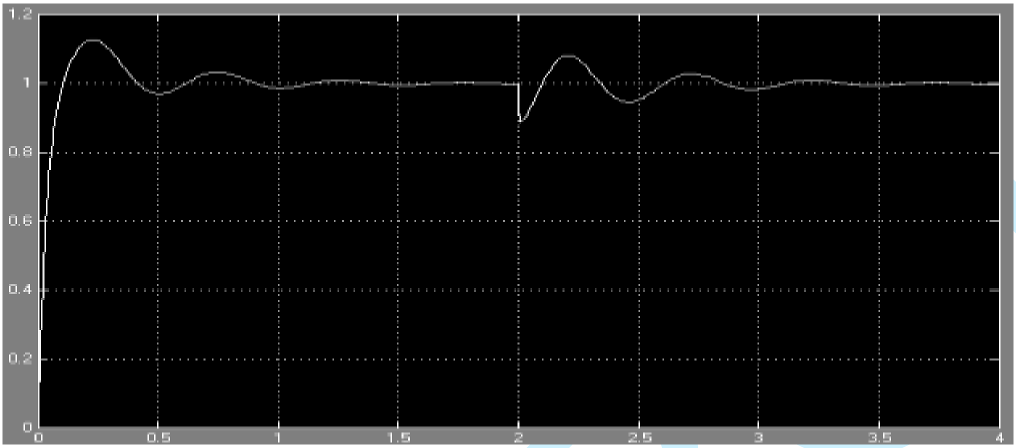

Figure 14. Output Of Disturbance Simulation

IV. CONCLUSION

It can be seen that our closed loop PID controller handles disturbances in the system similar to the way it handles a step function. The effect of any disturbance is minimized to within $\pm 12\%$ and stabilizes around 1 second after. The assignment is completed if it relaxes the design requirements. It is believe that the calculated values are correct and their responses should match. More control effort is required if the design was to continue and investigate a system with 5% overshoot. This would mean larger gains for our Kp, Kd, and Ki values. These values are already high and if we were to pursue more control it will not reflect the reality and limitations of PID controllers. It believes bus passengers will still feel comfortable with a 12% initial disturbance.

V. REFERENCES

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