Comparative Study of PID and Lyapunov-Based Adaptive Control for Dynamic Mobile Robot Trajectory Tracking

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Abstract*— Mobile robot control is an important topic in the field of control systems and robotics, particularly in dealing with parameter uncertainties and external disturbances. This study aims to analyze the performance of Lyapunov-based adaptive control compared to conventional PID for a mobile robot based on dynamic model. The evaluation is conducted using two types of trajectories, a circular trajectory representing smooth motion and a figure-eight trajectory representing complex motion with repeated changes in direction. The control method is designed using a cascade structure that integrates both kinematic and dynamic models. Simulation results show that both control methods are capable of tracking the reference trajectory and the adaptive control provides better performance with lower Root Mean Square Error (RMSE) values and faster convergence time. Furthermore, the adaptive control demonstrates superior capability in handling complex directional changes in the figure-eight trajectory. Therefore, the Lyapunov-based adaptive control is proven to be more effective and robust that the PID control in managing mobile robot with parameter uncertainties.

***Keywords*— Mobile Robot, Adaptive Control, PID, Trajectory Tracking, Dynamic Model, Lyapunov**

I. Introduction

Mobile robots are one of the rapidly growing areas in control systems and robotics with wide applications in autonomous transportation systems, service robots, and modern manufacturing industries. One of the main challenges in mobile robot control is the ability to perform accurate trajectory tracking [1], particularly when the system is subject to parameter uncertainties and external disturbances [2].

In the design of mobile robot control systems, two primary modeling approaches are commonly used, kinematic and dynamic models [3]. Kinematic models are widely used due to its simplicity and implementation, however it neglects important physical aspects such as forces, mass, and inertia. As a result, its applicability is limited in real-world scenarios where dynamic effects cannot be ignored [4]. In contrast, dynamic models provide a more realistic representation of the system behavior and more sophisticated for control strategies [5].

Various control methods have been developed to address trajectory tracking problem in mobile robots. Among them, the Proportional-Integral-Derivative (PID) controller remains one of the most used techniques due to its simplicity and ease of implementation. Despite its popularity, PID control heavily depends on parameter tuning and often exhibits degraded performance in the presence of system uncertainties and nonlinearities [6].

To overcome these limitations, adaptive control has emerged as promising alternative. Adaptive control techniques allow controller parameters to be adjusted online according to system variations, thereby improving robustness and tracking accuracy [7]. In particular, Lyapunov-based adaptive control has gained significant attention due to its strong theoretical foundation, which guarantees system stability while ensuring convergence of tracking errors [8]. Recent studies have demonstrated that adaptive control methods, including sliding mode [9] and neural-based approaches, can significantly enhance trajectory tracking performance in mobile robots under uncertain conditions [10].

In addition to control strategies, the complexity of the reference trajectory plays a crucial role in evaluating system performance. Many previous studies have focused on simple trajectories such as straight lines or circular paths, which are insufficient to fully assess the robustness of control algorithms [11]. In practical applications, mobile robot often operates in environments requiring complex and highly dynamic motion patterns.

Therefore, this study considers two types of trajectories, a circular trajectory and a figure-eight trajectory that introduces higher complexity due to repeated directional changes and intersection points [12].

The figure-eight trajectories are particularly challenging and provides a more rigorous benchmark for evaluating control performance under nonlinear and dynamic conditions [13].

This study aims to compare the performance of conventional PID control and Lyapunov-based adaptive control for a mobile robot using dynamic model. The control system is designed using a cascade structure that integrates both kinematic and dynamic model. Performance evaluation is conducted based on trajectory tracking, tracking error, and Root Mean Square Error (RMSE).

II. Research Methodology

A. System Modeling of the Mobile Robot

In this study, the mobile robot is modeled as a nonlinear system using both kinematic and dynamic approaches. The kinematic model describes the relationship between the robot's velocity and its position, while the dynamic model incorporates the effects od mass, inertia, and friction that providing a more realistic representation of the system [3].

The kinematic model of the mobile robot is defined as:

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \omega \quad (1)$$

where $\dot{x}$ and $\dot{y}$ denote the robot position, $\dot{\theta}$ is the orientation angle, v is the linear velocity, and ω is the angular velocity.

The dynamic model is expressed as:

$$m\dot{v} + b_v v = u_v$$
$$J\dot{\omega} + b_\omega \omega = u_\omega \quad (2)$$

where m is the robot mass, J is the moment of inertia, b_v and b_ω are friction coefficients, and u_v, u_ω are the control inputs.

B. Kinematic Controller Design

The kinematic controller is designed to generates reference linear and angular velocities based on the tracking error between the robot and desired trajectory. The tracking errors are defined in the robot's local coordinate frame as follows:

$$e_x = \cos\theta\,(x_r - x) + \sin\theta\,(y_r - y)$$
$$e_y = -\sin\theta\,(x_r - x) + \cos\theta\,(y_r - y)$$
$$e_\theta = \theta_r - \theta \quad (3)$$

The control laws are formulated as:

$$v_r = v_d\cos(e_\theta) + k_1 e_x$$
$$\omega_r = \omega_d + k_2 e_y + k_3\sin(e_\theta) \quad (4)$$

This approach ensures convergence of tracking errors for nonholonomic mobile robots [11].

C. PID Controller Design

The PID controller is employed as baseline method for comparison. It is used to regulate the robot's velocity to follow the reference signals that generated by the kinematic controller. The general form of the PID controller is given by:

$$u(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \frac{de(t)}{dt} \quad (5)$$

where $e(t)$ is the tracking error, K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain.

Lyapunov-Based Adaptive Controller Design

To address system uncertainties, a Lyapunov-based adaptive control strategy is implemented. This method ensures stability while allowing online estimation of unknown parameters such as mass and friction coefficients [2].

The adaptive control law is defined as:

$$u = \hat{m}a_r + \hat{b}v \quad (6)$$

where $\hat{m}$ and $\hat{b}$ respresent the estimated parameters.

The parameter adaptation laws are given by:

$$\dot{\hat{m}} = \gamma_m e a_r$$
$$\dot{\hat{b}} = \gamma_b e v \quad (7)$$

where γ_m and γ_b are positive adaptation gains.

This approach enables the system to adjust its parameters in real time, improving robustness and tracking performance under uncertain conditions [7].

D. Control System Architecture

The control process begins with the reference trajectory which defines the desired motion of the robot. This reference is compared with the actual position robot to generate a tracking error which serves as the primary input to the controller.

In the PID control system, the controller processes the error using fixed gains. The resulting control signal is applied to the robot dynamics to regulate its motion. While this approach is simple and widely used, its performance strongly depends on proper parameter tuning. Moreover, since the controller gains remain constant, the PID controller may struggle to maintain performance when the system experiences parameter uncertainties.

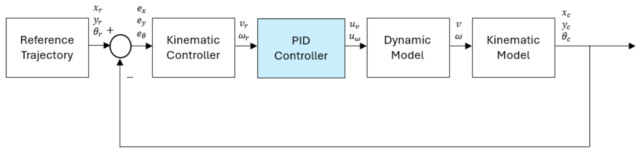

Figure 2. Diagram Block PID Controller

In contrast, the adaptive control system incorporates an additional adaptation mechanism that continuously updates the controller parameters based on the tracking error. Instead of relying on fixed gains, the adaptive controller adjusts its parameters in real time to compensate for variations in system dynamics, such as changes in mass and friction. This mechanism is typically designed using Lyapunov stability theory to ensure system stability and convergence of the tracking error.

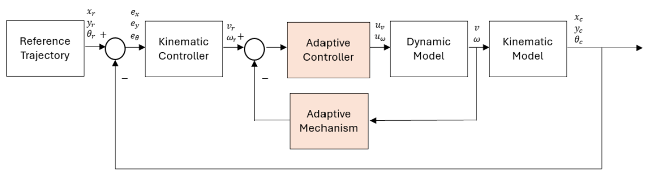

Figure 2. Diagram Block Adaptive Controller

The control signal generated by the adaptive controller is more flexible and responsive to system changes. As a result, the adaptive control system is able to achieve improved tracking accuracy, faster convergence, and better robustness compared to the PID controller, particularly in nonlinear and uncertain environments.

To evaluate the performance of the control strategies, two types of trajectories are considered, circular trajectory and figure-eight trajectory. To assess the performance of the control methods, several evaluation metrics are used, trajectory tracking, tracking error, adaptive parameters mechanism, and Root Mean Square Error (RMSE). These metrics provide a comprehensive evaluation of control performance in mobile robot systems.

III. Result and Discussion

The system parameters include the robot mass and friction coefficients which represent the dynamic characteristics of the robot and influence its motion response.

Table 1. Parameters Mobile Robot

Parameter	Symbol	Value
Robot Mass	m	5 kg
Friction Coefficient	b	2 kg/s

The parameters used in this study consist of system parameters, controller parameters, and adaptation gains that define the behavior of the mobile robot and its control performance. The PID controller parameters (K_p, K_i, K_d) are selected to regulate the tracking error by balancing responsiveness, steady-state accuracy, and damping behavior. In the adaptive control scheme, additional parameters such as the estimated mass and friction coefficients are introduced and updated online to compensate for uncertainties in the system dynamics. The adaptation gains determine the rate at which these estimated parameters are adjusted, affecting both the convergence speed and stability of the system.

Table 2. Control Parameters

Parameter	Symbol	Value
Estimated Mass	$\hat{m}$	3 kg (initial)
Estimated Friction	$\hat{b}$	1 kg/s (initial)
Proportional Gain (PID)	K_p	15
Integral Gain (PID)	K_i	1
Derivative Gain	K_d	8

Proportional Gain (Adaptive)	$K_{p,ad}$	10
Derivative Gain (Adaptive)	$K_{d,ad}$	5
Adaptation Gain (Mass)	γ_m	0.5
Adaptation Gain (Friction)	γ_b	0.5

The simulation results of the trajectory tracking performance using both PID and Lyapunov-based adaptive controllers. The evaluation is conducted on two types of trajectories, a circular and figure-eight paths to represent simple and complex motion scenarios.

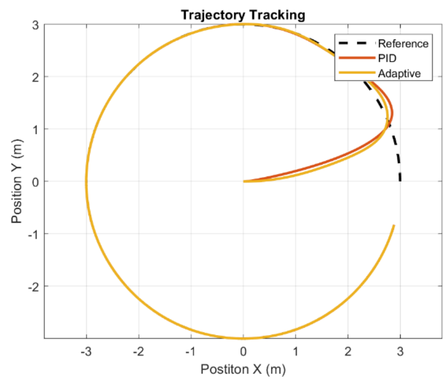

Figure 3. Circle Trajectory Tracking

Both control methods are able to follow the reference path with acceptable accuracy in the circle trajectory (Fig. 3). The adaptive controller produces a smoother trajectory that more closely to the reference and the PID controller exhibits small deviations and oscillations around reference path, particularly during the transient phase.

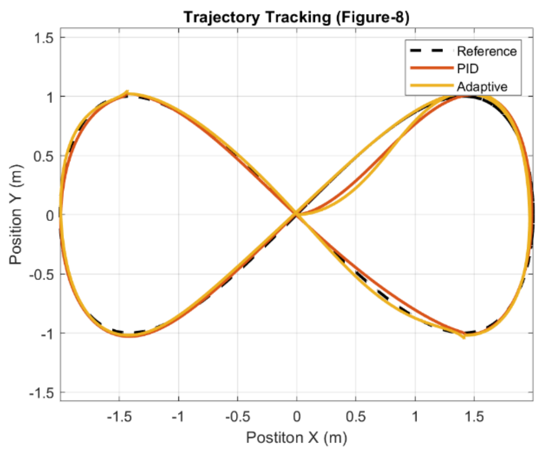

Figure 4. Figure-Eight Trajectory Tracking

In contrast, for the figure-eight trajectory (Fig. 4), which involves repeated directional changes and intersection points, the performance difference becomes more significant. The PID controller shows deviations near the intersection region where the trajectory curvature changes rapidly. Meanwhile, the adaptive controller maintains better tracking accuracy and stability, demonstrating its capability to handle complex nonlinear motion. These results indicate that trajectory complexity has a direct impact on control performance and adaptive controller provides superior tracking in more challenging conditions.

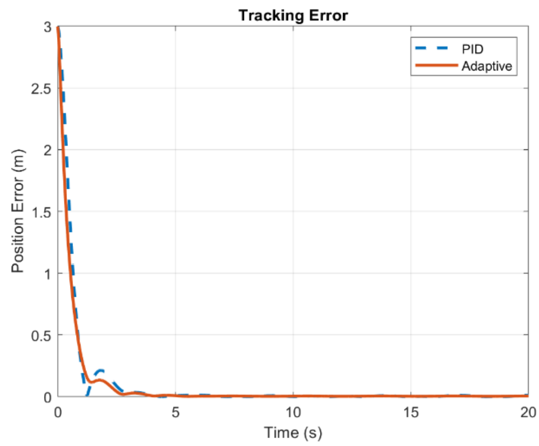

Figure 5. Tracking Error Circle Trajectory

In the circle trajectory, both controllers maintain low error values (Fig. 5), however the PID controller exhibits larger fluctuations compared to the adaptive controller.

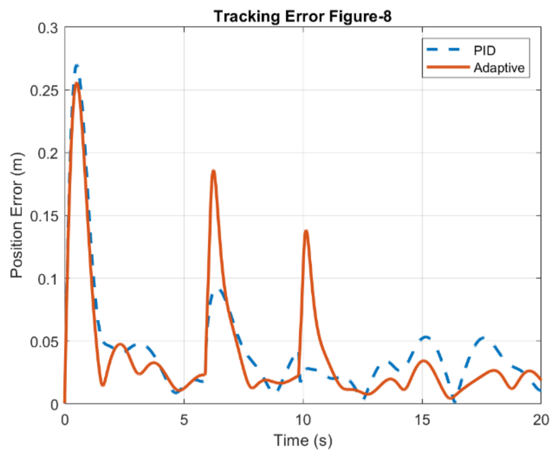

Figure 6. Tracking Error Figure-Eight Trajectory

The error magnitude of the PID controller increases significantly in the figure-eight trajectory (Fig. 6) where the robot must change direction rapidly. This behavior is due to the fixed-gain nature of the PID controller that limit its ability to adapt to varying system dynamics.

The adaptive controller demonstrates a more stable error with lower peak values (Fig. 6). Although minor fluctuations are still observed, the overall error remains consistently lower than the PID controller. This confirms that the adaptive mechanism effectively compensates for system uncertainties.

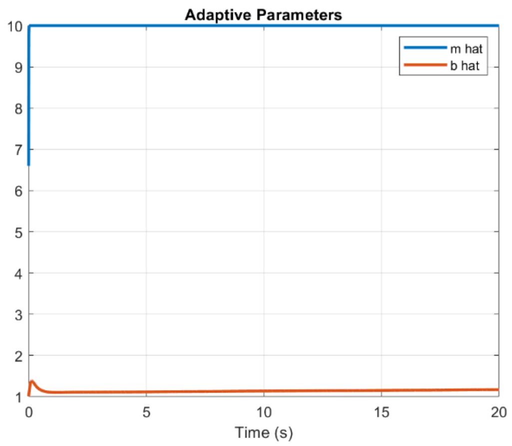

Figure 7. Adaptive Parameters Circle Trajectory

The evolution of estimated parameters in adaptive controller is also analyzed. The results show (Fig. 7) that the estimated parameters initially undergo significant changes as the controller adapts to the system dynamics and overtime these parameters converge to stable values.

During the figure-eight trajectory, small fluctuations in parameter estimates are observed, especially near trajectory intersections (Fig. 8). This indicates that the adaptation mechanism remains active and responsive to dynamic changes in the system. the convergence behavior confirms the effectiveness of the Lyapunov-based adaptation law in maintaining system stability.

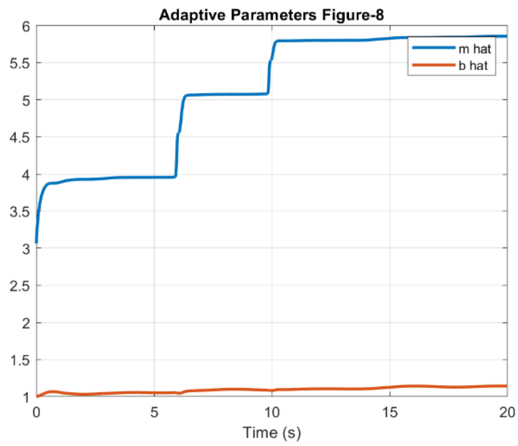

Figure 8. Adaptive Parameters Figure-Eight Trajectory

To quantitatively evaluate the tracking performance, the Root Mean Square Error (RMSE) is calculated for both controllers (Tab. 3). The results indicate that the adaptive controller consistently achieves lower RMSE values compared to the PID controller for both trajectories. The difference in RMSE becomes more pronounced in the figure-eight trajectory where the system experiences more complex motion. This demonstrates that the adaptive controller provides higher overall accuracy in trajectory tracking.

Table 3. RMSE Tracking

Controller	RMSE Circle Tracking (m)	RMSE Figure-Eight Tracking (m)
PID	0.4105	0.0624
Adaptive	0.3503	0.0625

IV. Conclusion

This study has presented a comparative analysis between conventional PID control and Lyapunov-based adaptive control for trajectory tracking of a mobile robot using a dynamic model. The control system was designed using a cascade structure that integrates both kinematic and dynamic model, and its performance was evaluated under two types of trajectories.

The simulation results show that both control methods are capable of tracking the desired trajectories and the adaptive controller approach consistently outperforms the PID controller in terms of tracking accuracy, convergence speed, and robustness. This is evidenced by lower RMSE values and reduced tracking error. The adaptive parameters estimation mechanism shows the effective convergence behavior, indicating the controller's ability to adjust to the system uncertainties in real time. This capability enhances system robustness compared to the fixed-gain controller.

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