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Inverse Kinematic of 1-DOF Robot Manipulator Using Sparse Identification of Nonlinear Systems

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***Abstract*— The Robot Manipulator is the most commonly used robot in the industry because it can imitate a human arm's ability to move objects. Extensive research has been conducted on robot manipulators, addressing various issues such as control systems, intelligent robots, degrees of freedom, mechanics-electronics systems, and other related problems. In the field of control systems, studies have focused on designing robot motion using kinematics. However, modeling the kinematic motion, which exhibits nonlinear characteristics, becomes increasingly challenging as the number of degrees of freedom increases. To overcome this challenge, this research proposes the use of sparse regression to model the kinematics of a robotic arm, employing the black box principle modeling. The obtained results demonstrate that the proposed method was capable of accurately identifying the robot manipulator, achieving a fitness score of up to 100%. This finding indicated that the proposed method can model the inverse kinematics of the manipulator robot without requiring complex calculations. This research was expected to provide further research opportunities in the area of kinematics identification using the proposed system identification method.**

***Keywords*—Robot Manipulator, Sparse Identification, Nonlinear, Kinematic**

I. Introduction

Robots is complex system and been invented since ancient age and on at that time it was still a mechanical system and then developed by adding hydraulic and pneumatic drive systems. Then since the 1950s robots begin to be equipped with electronic systems and start used in industry and until now robots growing and more widely applied in various fields [1]. Robotic manipulator that also known as a robotic arm is one type the most widely used robot in industry, since the robot used as a robot moving goods from one point to another and can also be used to perform other additional tasks [2]. Robot manipulator degrees of freedom is expressed in DOF, a manipulator robot can consist of 1-DOF to multiple DOF.

There are two mechanisms to modeling robot manipulator, namely dynamics modeling and modeling kinematics. Dynamics modeling is used to modeling robot dynamics systems by involving the influence of forces, moments and system and others environmental factors that affect the movement of the robot. Kinematics modeling is modeling that aims to move the robot relative to cartesian coordinates and local coordinates. There are two types of kinematics modeling namely forward kinematics modeling (direct) and inverse Kinematic. Direct kinematics is used to find coordinates cartesian through the coordinates of each of its joints known, while the inverse kinematics is used to find the angle value of each joint when the coordinate value of end effector is known or determined [3], [4]. This paper will focus on modeling robot kinematics which will be done by computer modeling.

There are several studies related to kinematics modeling robot, as was done in [5-8] which models kinematic inverse on the manipulator robot with using Denavit Hartenberg(D-H) matrix. This method uses matrices in its calculations and to model it requires quite a lot of effort high because it uses a mathematical equation with complex computing. Another solution in modeling kinematic equation is to use the approach geometric structure. This method is done with considering the position of the end effector relative to the position of the base, then do the calculations for each angle joint by using the trigonometry formulation and Pythagorean analysis. Several studies related to inverse kinematic modeling that uses a geometric approach as in [9-11].

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However, this method is quite difficult done if the number of DOF increases because it has to have good trigonometry analytical skills.

From some of these studies indicate that manipulator robot modeling needs adjustments if any differences in size, angle of motion and amount of DOF on the robot. However, when there are differences in the form and systematics in the manipulator robot causes the occurrence difference in kinematic equation. This research will propose the SINDy method used to model the kinematics of 1-DOF manipulator robot. From this research is expected to provide research opportunities related to kinematic identification systems on robots manipulator based identification system for degrees higher freedom.

II. Research Methodology

Kinematics modeling which is one of two modeling on the robot manipulator is modeling that does not consider the dynamics which affect the process of robot motion. Kinematics on 1-DOF robot manipulator is a kinematics equation which is basically easy to obtain, namely by using simple trigonometry formulas. However, the purpose of this research is to find the opportunities for research with a higher amount of DOF with using the proposed method. In this research the 1-DOF manipulator robot will be identified by its kinematics as shown in Figure 1.

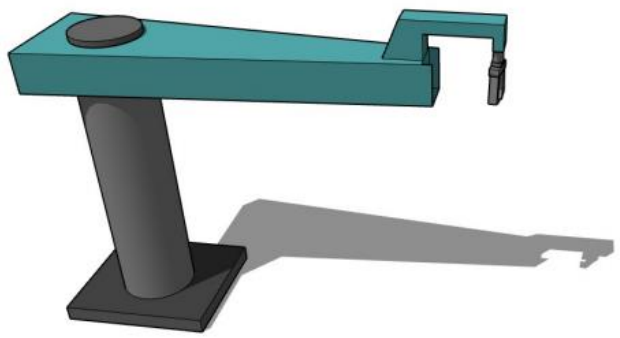

Figure 1. Design of Robot Manipulator 1-DOF

Kinematic formulation using the geometric approach can be obtained in the following which is shown in Figure 2. From the picture, then to get the model kinematic inverse can be done easily by using it can be seen that the kinematics is advanced on the 1-DOF robot shown in (1) and (2).

$$x = l * \sin(\theta) \quad (1)$$

$$y = l * \cos(\theta) \quad (2)$$

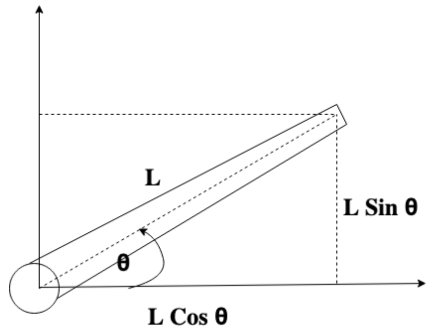

Figure 2. Kinematic 1-DOF

Meanwhile, when the position of the end effector is known or determined, the magnitude of the angle joints can be found using (3).

$$\theta = \tan^{-1}(y / x) \quad (3)$$

This kinematics modeling will be more complex when degrees of freedom increases. One approach that can be done is to use the black box modelling approach. Blackbox modeling is a modeling technique which uses only input and output measurement data and then is processed by using computer algorithm to generate mathematical equations that can be used as model of the robot kinematics. The computer algorithm used in this research was Sparse Identification of Nonlinear Dynamics (SINDy), the SINDy method was proposed for identification kinematics of 1-DOF manipulator robots and generate kinematic equations that can be used as kinematics inverse equation.

The SINDy method was proposed in 2016 by Brunton et al [12], this method is used to find similarities mathematics of the dynamics of non-linear systems. This method continues developed [13-15] and are increasingly being used on various applications. In this study, the SINDy method which was originally used for the identification of system dynamics will be proposed to find the mathematical equation of kinematics of a robotic arm with the same mechanism using input and output data then

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processed with SINDy. As for the similarities the mathematics of SINDy is shown in (4).

$$\theta = \Phi(X)\Lambda \tag{4}$$

where $\Lambda \in R^n$ is coefficient, X ∈ R^n is the input variable which can be the value of the position of the end effector (x or y) and

$$\Phi(X) = \begin{bmatrix} | & | & | & | & | & | & | \\ 1 & X & \ldots\cos^{-1}(X) & \sin^{-1}(X) & \tan^{-1}(X)\ldots \\ | & | & | & | & | & | & | \end{bmatrix} \tag{5}$$

The objective function of SINDy is to minimize value error stated in Equation 6.

$$e = \theta - \Phi(X)\Lambda \tag{6}$$

so to find the value of Λ then it can be with using the Least Square formula, which is as follows shown in (7).

$$\hat{\Lambda} = (\Phi(X)^T \Phi(X))^{-1} \Phi(X)\theta \tag{7}$$

III. Results and Discussion

This research is done by using simulation on the computer, by means of a simulation on forward kinematics with varying input θ values. This was done to ensure that the position value of the end effector matched the specified arm length. Then, the coordinate value of the end effector, which was the output of the forward kinematics block, became the input for the backward kinematics model. The output of the backward kinematics model was the value of θ. The measurement data is shown in table 1. Where X, Y are the end effector coordinate target, and θ is the degree of link.

Table 1. Measurement Data

X	Y	Θ
0	1	1.570
0.0991713838982745	0.995070367670398	1.471
0.193481209810633	0.981103980957275	1.376
0.278585739191426	0.960411362864459	1.288
0.351036712760017	0.936361696298306	1.212
...	...	...
...	...	...
-0.268668611097978	0.963232670444002	1.842

This test is carried out using the library consisting only of polynomial values up to the power of five, results obtained as shown in Figure 3. Where x0 is X and x1 is Y and x1’ is θ.

```
(x0)' = 1.000 x1
(x1)' = 2.712 1 + -1.354 x0 + -4.587 x1 + -0.733 x0^2 +
0.354 x0 x1 + 3.445 x1^2 + 2.644 x0^2 x1 + -0.760 x0^2
x1^2
fit score (%):

99.99999908453134
```

Figure 3. Result with Polynomial 5, Threshold=0.2

Figure 3 is the result of estimating the kinematic equation using SINDy and the Polynomial Library. The results showed that the Kinematics Equation for the 1 arm DOF could be expressed using a finite polynomial function of degree 2. The fitness value indicated that this function had a match of up to 99.99%. To verify this match, a simulation was conductaed to generate a graph using the same X and Y input values. The simulation results are depicted in Figure 4.

The subsequent test involved replacing the polynomial library used in Test 1 with inverse trigonometric functions. The purpose was to examine whether the proposed method could successfully identify and generate the kinematic equations, similar to the manual process illustrated in Equation (3). The test results using trigonometric functions are presented in Figure 6. It can be observed that the equation results obtained through the SINDy method align with Equation (3). Furthermore, the simulation test results, which employed varying inputs and produced the same outputs, are depicted in Figure 6.

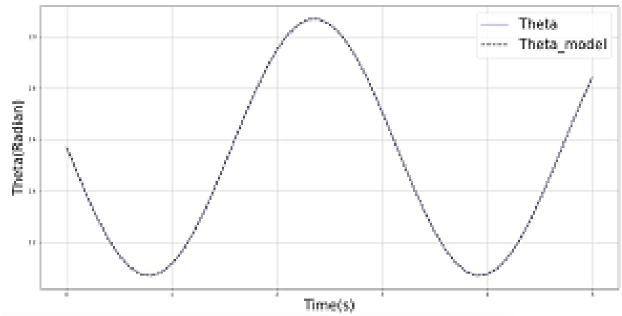

Figure 4. Simulation Results with Polynomial 5, Threshold=0.2

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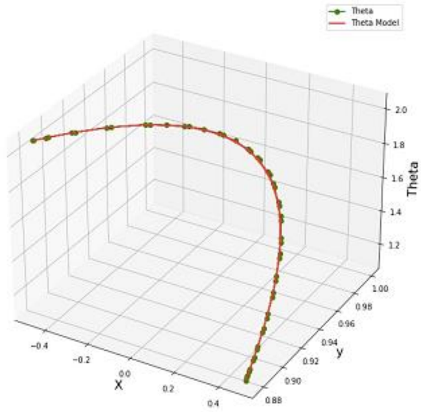

Figure 5. Simulation Results with Polynomial 5, Threshold=0.2

```
(x0)' = 1.000 *x1
(x1)' = 1.000 *np.arctan2(x1,x0)
fit score (%):

100.0
```

Figure 6. Equation result using Trigonometry Library

The figures 5 and 6 provide evidence that the SINDy method is capable of modeling the inverse kinematics equation using only the input data X and Y, along with the output data θ. These results demonstrated the effectiveness of trigonometry as a necessary library for robotic kinematics.

IV. Conclusion

1. By utilizing the polynomial library, the achieved results demonstrate an accuracy level approaching 100%. However, it should be noted that this accuracy might diminish as the number of degrees of freedom increases.

2. The examination conducted with the trigonometry function library yielded flawless outcomes, reaching a 100% match with the manually modeled results.

3. Based on these two outcomes, it can be inferred that SINDy can effectively identify the kinematics of the robot without necessitating manual modeling.

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